Sesión Especial 28

Teoría de grupos

Organizadores

- Yago Antolín Pichel (Universidad Autónoma de Madrid, ICMAT)
- Montserrat Casals Ruiz (Universidad del País Vasco/Euskal Herriko Unibertsitatea)
- Ilya Kazachkov (Universidad del País Vasco/Euskal Herriko Unibertsitatea)

Descripción

El objetivo de esta sesión es reunir investigadores de reconocido prestigio y jóvenes investigadores estudiando teoría de grupos desde distintos puntos de vista e interacciones con áreas como la Topología, a la teoría de representaciones, a la Geometría o a la teoría de la computación. La sesión será una oportunidad para compartir ideas entre los ponentes invitados y el resto de los participantes.

Programa

Lunes, 4 de febrero (mañana)

11:30 – 12:00	Antonio Viruel (Universidad de Málaga)
	On Lusternik-Schnirelman type invariants of groups
12:00 – 12:30	Dominik Gruber (ETH Zürich)
	Small cancellation theory over Burnside groups
12:30 – 13:00	Marialaura Nocce (Università di Salerno, UPV/EHU)
	Engel conditions in groups of automorphisms of rooted trees
13:00 – 13:30	Javier de la Nuez González (Universidad del País Vasco/Euskal Herriko Unibertsitatea)
	On the model theory of graph products of groups

Lunes, 4 de febrero (tarde)

17:00 – 17:30	María Dolores Pérez-Ramos (Universitat de València)
	Thompson-like characterization of solubility for products of groups
17:30 – 18:00	Carolina Vallejo Rodríguez (ICMAT)
	On character degrees of finite groups

18:00 – 18:30	Matteo Vannacci (Universität Düsseldorf)
	Probabilistic identities in profinite groups
18:30 – 19:00	Albert Garreta Fontelles (Universidad del País Vasco/Euskal Herriko Unibertsitatea)
	Systems of equations in groups and monoids

Systems of equations in groups and monoids

Albert Garreta Fontelles

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Abstract. I will survey and present the algorithmic problem of solving systems of equations in different algebraic structures, paying special attention to groups and monoids. I will then explain how such problem in many solvable groups and in certain one relator monoids is linked to some long-standing open problems in number theory and computer science, respectively.

This is joint work with Robert Gray and with Alexei Miasnikov and Denis Ovchinnikov.

Small cancellation theory over Burnside groups

Dominik Gruber

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Abstract. In 1902, W. Burnside asked whether every finitely generated group of finite exponent is finite. While this question has famously been resolved in the negative for large enough exponents (Novikov-Adian, Lysenok, Ivanov), constructing examples of infinite groups of finite exponent with additional prescribed properties has remained a notoriously hard task.

I will present a powerful new method for producing such examples that can be applied without any prior knowledge on groups of finite exponent. It is obtained by combining combinatorial and geometric interpretations of small cancellation theory, a theory that uses notions of negative curvature in groups to construct infinite groups with prescribed (and often exceptional) properties. As applications, we show the undecidability of Markov properties and the existence of Gromov's monsters in classes of groups of finite exponent, and we recover concise proofs of known results.

Joint work with Rémi Coulon.

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Engel conditions in groups of automorphisms of rooted trees

Marialaura Nocce

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Abstract. Groups of automorphisms of d-adic rooted trees ($Aut\mathcal{T}_d$) have been studied for years as an important source of groups with interesting properties. For example, many of their subgroups constitute a counterexample to the General Burnside Problem. The question whether every Engel group is locally nilpotent is the analogue of the general Burnside problem in the realm of Engel groups. Recall that an element x of a group G is said to be left Engel if for any $g \in G$ there exists an integer $n = n(g, x) \geq 1$ such that $[g,_n x] = 1$. We denote this set by $L(G)$. If $L(G) = G$ we say that G is an Engel group. In this talk, we introduce basic notions of the theory of groups of automorphisms acting on d-adic rooted trees, and then we prove that for any $d \geq 2$ the group $Aut\mathcal{T}_d$ has no nontrivial Engel elements. This is a consequence of a more general result: if G is the iterated wreath product of any infinite sequence of non-trivial finite groups, then $L(G) = 1$.

Joint work with G. Tracey and G. Traustason.

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On the model theory of graph products of groups

Javier de la Nuez González

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Abstract. When studying the model theory of groups, it is natural to ask which group-theoretic constructions preserve the elementary theory. In 1959, Feferman and Vaught studied the first-order properties of direct products and showed, in particular, that the direct products of elementarily equivalent groups are elementarily equivalent. In contrast, invariance of the elementary equivalence for free products of groups was a long-standing conjecture which was recently solved by Sela (2017).

In this talk, we will center on the converse question: given two elementary equivalent free products of groups (or more generally, graph product of groups), when are the factors elementarily equivalent? We will discuss some sufficient conditions and use our results to describe finitely generated groups elementarily equivalent to chordal RAAGs.

Joint work with M. Casals-Ruiz, A. Garreta and I. Kazachkov

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Thompson-like characterization of solubility for products of groups

María Dolores Pérez-Ramos

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Abstract. A remarkable result of Thompson states that a finite group is soluble if and only if its two-generated subgroups are soluble. This result has been sharply generalized, and it is in the core of a wide area of study in the theory of groups, aiming for global properties of groups from local properties of two-generated (or more generally, n-generated) subgroups. We report about an extension of Thompson's theorem from the perspective of factorized groups. We prove that for a finite group $G = AB$, with A, B subgroups of G, if $\langle a, b \rangle$ is soluble for all $a \in A$ and all $b \in B$, then $[A, B]$ is soluble. In that case, the group G is said to be an $\mathcal{S}$-*connected product* of the subgroups A and B, for the class $\mathcal{S}$ of all finite soluble groups. As an application, deep results about connected products of finite soluble groups, for other relevant classes of groups, are extended to the finite universe.

On character degrees of finite groups

Carolina Vallejo Rodríguez

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Abstract. I will discuss some results that show the influence of group invariants related to character degrees on the group structure.

Joint work with Eugenio Giannelli and Mandi Schaeffer-Fry.

Probabilistic identities in profinite groups

Matteo Vannacci

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Abstract. A non-trivial element $w = w(x_1, \ldots, x_k) \in F_k$ is said to be an *identity* in a group G if $w(g_1, \ldots, g_k) = 1$ for every $g_1, \ldots, g_k \in G$. Examples of such groups are nilpotent groups of bounded class and solvable groups of bounded length, and groups satisfying identities have been extensively studied. On the other hand, when many finite quotients of the group are available, one might want to keep track of the behaviour of identities on the finite quotients. A non-trivial word w is said to be a *probabilistic identity* in a residually finite group G if there exists $\delta > 0$ such that in any finite quotient Q of G the probability that w is satisfied in Q is bounded below by δ; the infimum of this quantities will be denoted by $P(G, w)$. This concept can be handled with profinite techniques.

Clearly, identities are probabilistic identities (with $P(G, w) = 1$) and Larsen-Shalev made the surprising conjecture that also a sort of converse should hold, namely: if a finitely generated residually finite group satisfies a probabilistic identity, then it satisfies some identity. This conjecture has been confirmed for many classes of groups, such as linear groups, weakly branch groups and groups with "many" non-abelian composition factors. In this talk I will present some positive evidence of this conjecture by studying the functions $f_k : F_k \to [0, 1]$ defined by $f_k(w) = P(G, w)$.

On Lusternik-Schnirelman type invariants of groups

Antonio Viruel

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Abstract. Following the ideas introduced by Eilenberg-Ganea in their seminal work [Annals of Math, 1957], we study Lusternik-Schnirelman type invariants associated to morphisms bettween groups: every group morphism gives rise to a continous maps bettween classifying spaces, and the homotopy invariants of this map become invariants of the original group morphism. So, within the spirit of Eilenberg-Ganea, we shall introduce cohomological invariants that mimic Lusternik-Schnirelman type invariants induced by classifying spaces as described above.

Joint work with Zbigniew Błaszczyk, José Gabriel Carrasquel-Vera and Arturo Espinosa Baro.

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